Transient Response of Ring-Stiffened Thin Cylindrical Shell with Symmetric Mass Loading

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Theme

THE transient response of simply-supported, circular, cylindrical, thin-shell structures is considered. The effect of ring stiffeners and concentrated masses is analyzed in detail. Superposition of normal modes is used to compute the dynamic deflection and stress resultants for two transient surface loadings, a) an axial band of uniform pressure around the circumference and b) a plane pressure wave which approaches and passes the shell in a direction normal to its axis of symmetry. Numerical results are presented in graphical form for each surface loading for unstiffened and stiffened mass-loaded shells.

Contents

The ring-stiffened-shell-structure of a missile, a spacecraft, or a submarine may be subject to an external blast wave. Thus, in their design it is necessary that the response of such a structure in-vacuo and/or in a fluid medium to a transient pressure pulse is known.

Prior work dealing with the response of finite shells to impulsive loads has dealt largely with unstiffened shells such as that by Sheng,[1] Cottis,[2] Bothel and Hubka,[3] Liao and Kessel.[4] The response of a finite, ring-stiffened shell with concentrated masses attached to the stiffener was studied by Michalopoulos and Muster.[5,6] The plane-symmetric locations of the masses in their problem is similar to those where machinery and equipment are fastened to a submarine hull. In Ref. 5 and 6, they obtained the natural frequencies and normal modes of vibration of a thin circular-cylindrical shell by the Rayleigh-Ritz method. The shell was ring-stiffened and mass-loaded. The purpose of the study summarized here[7] is to obtain by modal superposition the response of the shell structures described in Fig. 1.

Two transient surface loadings are considered: a circumferential band of uniform radial pressure applied as a time pulse (Fig. 1), and a plane pressure wave passing over the cylinder in a direction normal to the cylinder axis (Fig. 2).

By superposition of modal solutions, dynamic deflections, membrane loads, and moments are obtained for an unstiffened, a ring-stiffened, and a ring-stiffened mass-loaded shell for the pressure-band and plane-wave surface loadings.

The matrix form of the equations of motion of a conservative multi-degree of freedom system in forced vibration is

$$[M]\{\ddot{S}\} + [K]\{S\} = \{Q\}, \tag{1}$$

where $[M]$ and $[K]$ are the mass and stiffness matrices, respectively; $\{S\}$ is a set of generalized coordinates, that is, the vector whose elements are the time-dependent amplitudes $(\bar{C}_{mn}, \bar{B}_{mn}, \bar{A}_{mn})$ of the displacements (u, v, w), respectively; $\{Q\}$ is the vector of generalized forces representing the loads applied to the shell (assumed functions of time only).

The displacements (u, v, w), whose positive direction is shown in Fig. 1, are determined from the solution of (1), $\{S\}$, by means of

$$\begin{aligned} u &= \sum_{m=1}^{M} \sum_{n=0}^{N} \bar{C}_{mn}(t) \cos(n\theta) \cos(m\pi\alpha) \\ v &= \sum_{m=1}^{M} \sum_{n=0}^{N} \bar{B}_{mn}(t) \sin(n\theta) \sin(m\pi\alpha) \\ w &= \sum_{m=1}^{M} \sum_{n=0}^{N} \bar{A}_{mn}(t) \cos(n\theta) \sin(m\pi\alpha) \end{aligned} \tag{2}$$

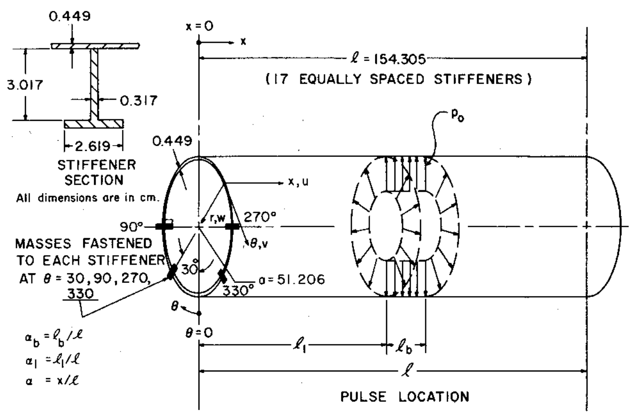

Fig. 1 Shell geometry, coordinate system and pressure-band load.

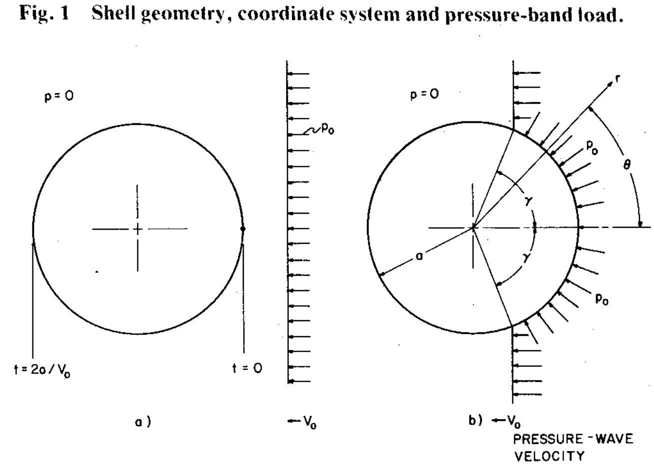

Fig. 2 Plane-wave load.

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A solution of (1) can be found by means of the normal coordinate transformation

$$\{S\}=[\varphi]\ \{q\} \tag{3}$$

where $[\varphi]$ the modal matrix, is the square matrix whose columns are the eigenvectors of the free vibration problem, and $\{q\}$ is a set of normal coordinates. It can be shown[7] that the solution of Eq. (1) is

$$\{S(t)\}=\sum_{i=1}^{NF}\{\varphi\}_i\{\varphi\}_i^T\int_0^t\frac{\sin\omega_i(t-\tau)}{\omega_i}\{Q(\tau)\}\mathrm{d}\tau$$
$$+\sum_{i=1}^{NF}\{\varphi\}_i\{\varphi\}_i^T[M]\left[\{S(0)\}\cos\omega_i t+\{\dot{S}(0)\}\frac{\sin\omega_i t}{\omega_i}\right] \tag{4}$$

where N_F = the number of degrees of freedom and the generalized forces Q_{mn} are given by

$$Q_{mn}=a\ell\int_0^{2\pi}\int_0^{\ell}p(x,\theta,t)\cos(n\theta)\sin(m\pi\alpha)\mathrm{d}\alpha\mathrm{d}\theta \tag{5}$$

where $p(x, \theta, t)$ describes the pressure wave on the cylinder, m is the number of axial half-waves, and n, the number of circumferential waves.

Membrane loads and shell moments may be obtained by using the magnitudes of the displacements u, v, w ($\bar{C}_{mn}$, $\bar{B}_{mn}$, and $\bar{A}_{mn}$, respectively) and the results in Ref. 5, that is

$$N_x=\frac{Eh}{a(1-\nu^2)}\sum_{m=1}^{M}\sum_{n=0}^{N}\Big[\nu(-\bar{A}_{mn}+n\bar{B}_{mn})-\frac{m\pi a}{\ell}\bar{C}_{mn}\Big]\cdot\cos(n\theta)\sin\left[\frac{m\pi x}{L}\right]$$

$$N_\theta=\frac{Eh}{a(1-\nu^2)}\sum_{m=1}^{M}\sum_{n=0}^{N}\Big[-\bar{A}_{mn}+n\bar{B}_{mn}-\frac{\nu m\pi a}{\ell}\bar{C}_{mn}\Big]\cdot\cos(n\theta)\sin(m\pi\alpha)$$

$$N_{x\theta}=\frac{Eh}{2(1-\nu)}\sum_{m=1}^{M}\sum_{n=0}^{N}\left[\frac{m\pi}{L}\bar{B}_{mn}-\frac{n}{a}\bar{C}_{mn}\right]\cdot\sin(n\theta)\cos(m\pi\alpha)$$

$$M_x=D\sum_{m=1}^{M}\sum_{n=0}^{N}\left\{\left[\left(\frac{m\pi}{L}\right)^2+\nu\left(\frac{n}{a}\right)^2\right]\times\bar{A}_{mn}-\nu\frac{n}{a^2}\bar{B}_{mn}\right\}\cdot\cos(n\theta)\sin(m\pi\alpha)$$

$$M_\theta=D\sum_{m=1}^{M}\sum_{n=0}^{N}\left\{\left[\left(\frac{n}{a}\right)^2+\nu\left(\frac{m\pi}{L}\right)^2\right]\times\bar{A}_{mn}-\frac{n}{a^2}\bar{B}_{mn}\right\}\cdot\cos(n\theta)\sin(m\pi\alpha)$$

$$M_{x\theta}=\frac{D(1-\nu)}{a}\sum_{m=1}^{M}\sum_{n=0}^{N}\left[-\frac{nm\pi}{L}\bar{A}_{mn}+\frac{m\pi}{L}\bar{B}_{mn}\right]\cdot\sin(n\theta)\cos(m\pi\alpha)$$

From few examples it is difficult to generalize about the dynamic response of structures. However, we observed certain characteristics of the differences in response between: 1) a finite, unstiffened shell, 2) a finite, ring-stiffened shell, and 3) a finite, ring-stiffened shell with concentrated mass loads and arranged in a plane symmetric manner.

Concerning the response of the unstiffened and stiffened shells (in terms of their radial displacements) to a centrally positioned pressure-band load: during the time the pulse is applied, the stiffened-shell response was less (in amplitude) than the unstiffened-shell and slightly greater in frequency, suggesting that the additional mass of the stiffeners has little relative effect. The response at the mid-line of the shell decreases with time as the energy moves to the shell ends (Figs. 6 and 7, Ref. 7). The frequency of the response of the stiffened shell at the intersection of the symmetric plane with the shell decreases as the size of the massloads is increased (Figs. 8 and 9, Ref. 7). The response of an unstiffened and a stiffened shell can be computed with reasonable accuracy by superimposing the response of relatively few modes (Figs. 12 and 13, Ref. 7).

Concerning the response of the shells to a plane-wave loading: the maximum response (in terms of the radial displacements) is significantly greater for a plane-wave load than for a pressure-band load of the same magnitude (that is, p_0 is the same for both cases) (Fig. 14, Ref. 7).

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